Reflections on Parity Nonconservation*

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This paper considers the implications for the relational-substantival debate of observations of parity nonconservation in weak interactions, a much neglected topic. It is argued that 'geometric proofs' of absolute space, first proposed by Kant (1768), fail, but that parity violating laws allow 'mechanical proofs', like Newton's laws. Parity violating laws are explained and arguments analogous to those of Newton's *Scholium* are constructed to show that they require absolute spacetime structure—namely, an orientation—as Newtonian mechanics requires affine structure. Finally, it is considered how standard relationist responses to Newton's argument might respond to the new challenge of parity nonconservation.

1. Introduction. Much has been written concerning chirality and especially concerning its connection to the substantivalist-relationist debate (e.g., Van Cleve and Frederick 1991), but almost all has followed Kant (1768) in considering the challenge to be that of giving a relationist account of geometric properties. A surprisingly small amount of this work has considered parity violating natural phenomena, and almost nothing at all (a notable exception and an inspiration for this essay being Earman 1989, 147–150) has been written about the problems faced by the relationist in formulating laws for such phenomena (forthcoming work by Carl Hoeffer and Simon Saunders, as well as this piece, seeks to fill this gap). Given the

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considerable attention directed to the question of how Newton's mechanical laws bear on the issue, this neglect is remarkable. In this paper we will explore carefully the character of parity violating laws and discuss whether the relationist has the resources to formulate them.

We will follow the following plan: in the first two sections we will review the relationist-substantivalist debate, highlighting important aspects concerning the problem of inertia and Kant 1768. Then we will consider the phenomena that reveal parity violating effects, and the theories that describe them, and introduce a useful toy theory in §3–4. In the remainder of the paper we will consider how the substantivalist can motivate an argument against relationism based on parity nonconservation that is directly analogous to Newton's arguments from inertia (§5) and how a relationist might go about constructing a suitable theory (§6).

There are several goals to this exercise. First, and most obviously, we wish to see whether parity nonconservation adds any interesting new arguments to the relationist-substantivalist debate; the contention of this paper is that the new arguments are exactly parallel to existing ones. Second, this paper aims to provide a useful account of parity violating theories to aid philosophers in extending the debate. And finally, more subtly but equally importantly, by drawing analogies between Newton's arguments and those involving parity violation, new light will be shed on the role of absolute spacetime structures in physics and in arguments concerning space.

2. Newton's Argument. Consider two familiar stances regarding spacetime (both paraphrasing Earman 1989, 114):

Substantivalism: "both bodies and space are substances in that bodies and space points and regions are elements of the domains of the intended models of [our most comprehensive theories] of the physical world."

Relationism: only bodies are substances in that only bodies are elements of the domains of the intended models of our most comprehensive theories of the physical world.

A vindication of either stance requires an appropriate theory that comprehends the important phenomena; in this paper the relevant phenomenon is that of parity nonconservation (PNC), but to introduce some important ideas consider first the more familiar phenomena surrounding the 'problem of motion'. Let us take it that considerations such as Newton's bucket experiment (Newton 1686, 6–12) and observed effects of rotations show that motions exhibiting identical changes in local relations can yet be distinguished by inertial effects. The contemporary (classical) substantivalist proposes Newtonian mechanics formulated in Galilean spacetime—call the adoption of this theory 'Newtonianism'—vindicating substantivalism. The broad idea is that one can explain the phenomena only by laws that invoke some 'absolute structure' of spacetime, which in turn fosters substantivalism.

The relationist cannot respond with a theory in which inertia is determined by purely local relations, and her position is made even harder by Newton's second thought-experiment in which a pair of globes connected by a rigid rod rotates about its center of mass in an otherwise empty universe. I have argued (Huggett 1999a) for the following formulation of the globes argument.

No theory of motion is:

(a) Leibniz relational, having models only with elements representing material points related by primitive temporal and instantaneous spatial separations;
(b) *and* theoretically complete with respect to Newtonianism, so that there is a 1:1 mapping between its dynamically possible models and those of Newtonianism (up to diffeomorphism) that preserves all relations, forces, and masses;
(c) *and* such that inertial effects represented by the dynamically possible models supervene on the relations between bodies in the models.

Proof: since Newtonian mechanics treats every possible rate of rotation as dynamically distinct, so does the theory by (b), so by (c) the relations instantiated by the models must differ, contrary to (a), given that the globes exhaust the matter content of the universe.

The relationist is then faced with giving up one of three appealing propositions: the adequacy of the Leibnizian relations, power equal to that of the most successful known theory in the field, or the idea that inertial effects correlate with (are 'due to') relative motions of bodies. Relationist theories can then be categorized according to their responses to Newton's challenge: For example, Sklar (1974, §III.F) and Maudlin (1993) consider expanding the set of primitive spatial predicates; Machian theories (Mach 1883, 279–296) give up on completeness, usually invoking skepticism about Newtonian models radically different from our world; and van Fraassen (1970, §IV.1) denies supervenience. All three approaches will be used later to analyze the new problems of motion raised by PNC, and since Sklar and van Fraassen are probably less familiar, we will briefly develop their suggestions here (for details see Huggett 1999a).

The key to both schemes is the idea of an *adapted relative reference frame* (Friedman 1983, 223–226): given four suitable points of a rigid body, one can construct a system of Cartesian space and time coordinates rigidly attached to the body so that one point represents the origin and the others represent points on orthogonal axes, and so that the Euclidean coordinate distance between any two bodies equals their Leibnizian separation. Since this construction is given entirely in terms of relations to the reference body, we shall take it without further discussion that such frames are relationally permissible (they are Newton's 'relative spaces').

Nothing in this construction supposes that the frame is inertial, and indeed, as Friedman argues (1983, §III.7), there is no guarantee that at a given moment any reference body will have inertial motion. In response we can use Sklar's idea that 'absolute acceleration' might not be a kind of motion relative to anything—bodies or space—at all, but rather a monadic property, represented by a triple of real numbers, $s_f(t)$, in any reference frame, f. The difference of the monadic accelerations of two bodies is always equal to their relative acceleration, and the absolute acceleration, S, and angular velocity, ω, of any frame are determined by the monadic accelerations of the four reference points. Then the Sklarian theory of mechanics states that in a given frame with applied force, **F**, the motion, **r**(t), of a body is governed by the generalization of Newton's second law:

$$d^2\mathbf{r}/dt^2 = \mathbf{F}/m - \mathbf{S} - 2 \cdot \omega \times d\mathbf{r}/dt - d\omega/dt \times \mathbf{r} - \omega \times (\omega \times \mathbf{r}). \quad (1)$$

The equation is of course 'ripped off' from Newton, but arguably this theory does not involve a 'trivializing instrumentalism' (contrary to Earman 1989, 126–128). In Huggett 1999a I argue, in particular, that such theories do not collapse into instrumentalism according to any of the characterizations of instrumentalism given by Earman, and that they are clearly opposed to 'manifold substantivalism' with its strong commitments to the reality of spacetime points as individuals. Theories like this can be understood as reinterpreting Newtonian mechanics in terms that do not hypostatize space.

Van Fraassen's approach is related, but starts from the observation that relative to any reference body one can construct arbitrary coordinate systems, where the new coordinates are arbitrary smooth functions of the adapted coordinates. In particular, there are those which correspond to arbitrary translations and changes in motion of the reference body, including inertial frames. Van Fraassen's theory thus states that 'there are frames in which $d^2\mathbf{r}/dt^2 = F/m$ holds'. This theory is not given axiomatically, but in the semantic view of theories, to which van Fraassen, and seemingly most participants to the debate, subscribe, it is a scientific theory, because it specifies a collection of models.

We will not question further here the adequacy of these approaches to mechanics, but will assume that they are viable, and hence use them later to study the prospects for a relationist account of PNC.

3. Geometric Challenges. In his essay, Kant (1768, 28) distinguishes his 'geometric' proof of absolute space from Euler's (1748) 'mechanical' proof, which—like our reading of Newton—"only brings to view the difficulty of assigning to the most general laws of motion a determinate meaning, should we assume no other concept of space than that obtained by abstraction from the relation of actual things." Kant's distinction can be understood through the two ways in which relationist theories might fail to be suitably comprehensive of the phenomena: first, as Newton and Euler argue, a theory might fail to yield an adequate account of some mechanical process, such as the observed occurrences of inertial effects; and second, some possible state of affairs might not be describable in terms of certain given relations, and in particular some ostensibly spatial property might not reduce to the primitive relations, as, for instance, metric relations in Lorentzian spacetimes are not determined by relations of causal connectability. Thus, for example, the relationist has no trouble *representing* the different rotational states of Newton's bucket because different rotations are distinguished by the relative heights of the water, but she does have trouble *theorizing* the different rotations. The mechanical proofs thus challenge the theoretical adequacy of relationism, whereas Kant's geometric proof is envisaged as an attack on representational adequacy.

However, there is fairly wide agreement (Van Cleve and Frederick 1991) that substantivalism does not score over relationism in the way that Kant suggested in 1768, though an important hole in the relationist account remains. This paper thus addresses the question of whether a 'mechanical' or 'theoretical adequacy' challenge concerning chirality and PNC—parallel to that concerning inertia—is more decisive. First, however, we should briefly consider the relationist response to the geometric challenge to avoid later confusion about the relationist interpretation of various chiral concepts. Though we slice things somewhat differently, our discussion broadly follows Earman 1989, Ch. 7, and the reader is referred there for a more thorough explanation of the relationist position.

Much has been written about Kant's (1768) intentions, so we will not attempt a detailed discussion of them here. All we require is his central observation that mirror images are exactly alike in internal relations and yet certain mirror images are distinct in that they cannot be made to exactly coincide. Since this distinction seems to be spatial, but does not correlate with any difference in internal relations, it appears that there is a representational problem for the relationist: how is the difference to be described in relational terms? Actually, the topic of mirror symmetry involves a family of spatial concepts all of which present challenges to the relationist; we shall address Kant's challenge with relational accounts of the various aspects of chirality.

(i) Bodies whose mirror images cannot be superimposed are enantiomorphs, and they are characterized in geometry by the absence of a plane of (reflection) symmetry. An appropriate relational definition thus states that 'a body in an n-dimensional relational space of constant curvature is an *enantiomorph* if when embedded in a metric space of the same dimension and curvature it has no (n – 1)-planes of symmetry in that space'. The restriction to constant curvature is necessary because the idea of a reflection or a plane of symmetry is only generalizable from Euclidean geometry in that case (a point that applies equally to the substantivalist). Since we assume constant curvature for the space (or at least approximately for the region in which the body lies) the curvature can be determined from the metrical relations of the points of the body (assuming it has an open n-dimensional part). Finally, the relationist only denies that physical space exists beyond the relations of bodies, not that mathematical spaces exist, so she may legitimately talk about embeddings in such abstract spaces.

(ii) The concept that seems closest to Kant's concerns is that of having a certain handedness or chirality: of being, say, left rather than right. Now, mirror images may have identical internal relations, but, if they are enantiomorphs, they will not have identical relations to external (enantiomorphic) bodies: e.g., my left hand, but not my right hand, can bear the relation of coincidence to a left-handed glove. So the natural relationist definition states that 'the *chirality* of a body is the set of relations that it can, but which its mirror image cannot, bear to other objects'. Such accounts are developed by Huggett (1999b, Ch. 11) and Gardner (1990, Ch. 17). Of course, implicit in the definition is that a chirality depends on the existence of external bodies (enantiomorphs at that), so that a hand alone in a universe—as envisioned by Kant—has no determinate handedness. Kant attempted to show that this conclusion is an absurdity, but he is generally judged to have failed.

Note the conventionality if we construe the property of being of a determinate handedness as the property of being left-rather-than-right (or vice versa). For we have identified the different chiralities with the sets of possible relations, but it is conventional whether we call a set 'left' or 'right'; in some cases—e.g., gloves—there is a natural choice about which is left given a left hand, in others—e.g., screws—it is more obviously arbitrary. The interesting point is that one could have a language in which there were no terms distinguishing left and right, and yet we could still speak of those bodies that bore external relations different from their mirror images: objects could—in a sense—be handed without being (called) left or right.

(iii) In the Euclidean plane, an enantiomorph and its mirror image are incongruent counterparts: related by a reflection, yet geometrically distinct since they cannot be made to coincide. However, as is well known, whether two bodies are incongruent counterparts depends not just on their being enantiomorphs and mirror images, but also on the space in which they are embedded being orientable: for example, the symbols 'F' and 'ꟻ' are incongruent in the plane, but not on a Möbius strip. Thus the relationist can define that 'two bodies are *incongruent counterparts* if they are mirror image enantiomorphs embedded in an orientable space (of constant curvature)'. Clearly this definition is only satisfactory insofar as the relationist can give a definition of an orientable space.

(iv) In differential topology, an n-dimensional space is orientable just in case it admits a nowhere vanishing n-form (e.g., Baez and Munian 1994, 82–87). Equivalently, it has a covering of open sets, $\{U_i\}$ such that every set admits coordinates of the same orientation: on the intersection of any two sets U and U' with coordinates $x^\mu(p)$ and $y^\nu(p)$ respectively, $\det(\partial x^\mu/\partial y^\nu) > 0$. This means that the axes which each chart in the covering—i.e., pair, $[U, x^\mu(p)]$—pulls back to the space from $\mathfrak{R}^n$ have the same handedness wherever they overlap.[1]

Now, this definition, since it makes reference to differential forms (or mappings from a space) requires that the reality of the manifold be taken seriously, and thus the relationist cannot take these definitions to characterize orientability for physical space, because she denies the reality of physical space as a manifold. Of course, Kant was unaware of the possibility that space might have a topology other than that of $\mathfrak{R}^3$, so whatever the substance of his geometric challenge, it was not (intentionally) that the relationist define orientability (though see Nerlich 1976, Ch. 2). However, his arguments have recently lead a number of commentators to this point because the job of giving a general representation theorem of the form 'space S is *orientable* iff relations of type ____ are instantiated' seems overwhelming—but apparently necessary for a definition of orientability in relational terms. In response, Brighouse (1999) proposes (more or less) that 'a space is *orientable* if mirror images embedded in the space cannot be made to coincide by any possible rigid motions', allowing the relationist possible as well as actual relations. Unfortunately, this definition invokes rigid motions and so only holds in spaces of constant curvature, whereas orientability is a far more general notion. Thus it would be preferable if a more general account could be found (one that makes the existence of incongruent counterparts a consequence of orientability, not constituitive of it). What such an account might be we will not discuss here, though the question is under active research.

1. Incidentally, it does not follow that there are n nowhere vanishing, everywhere linearly independent vector fields—n-ads—representing a handedness to space, as Earman (1989, 141) suggests; some orientable spaces, such as the sphere, do not admit even one nowhere vanishing vector field. This is not a significant point, but worth mentioning as I have seen Earman's definition repeated elsewhere.

In what follows we shall assume that space is orientable (and flat, given the equations of motion), so we will not worry further about this final problem, and take all the concepts to be defined for both substantivalist and relationist. Our next steps then are to consider PNC phenomena and theories, and how these theories bear on the relationist-substantivalist debate.

4. PNC Phenomena and Theories. PNC is manifested in a number of experiments of which we will mention just one (see Commins 1993, for an up-to-date compendium, and Maglich 1973, for a wonderful collection of commentaries, reminiscences, and early papers). Behind them all, though, is the observation by Lee and Yang (1956) that one must choose carefully which quantities to measure if one wishes to test for PNC—which we shall understand for the present to mean that some natural processes occur with a certain handedness as a matter of natural law. Especially, if one measures scalar products of vector quantities, these will, by definition, agree for a process and its mirror image: since vectors $\vec{V} \rightarrow -\vec{V}$ under reflection. Intuitively, vector lengths and angles are preserved by a reflection, so the mirror image of a pair of vectors will be a combination of rotations and translations, corresponding simply to a relocation in space of the process. One must instead measure the scalar product of vector and axial-vector quantities; since axial-vectors transform $\vec{A} \rightarrow +\vec{A}$, $\vec{V} \cdot \vec{A} \rightarrow -\vec{V} \cdot \vec{A}$—a pseudoscalar quantity. Intuitively, any combination of rotations and translations that aligns a vector with its mirror image will place an axial-vector in anti-alignment with its mirror image, and so the vector-axial-vector scalar product will differ from that of its mirror image (unless it vanishes). Thus, if one product is physically necessary, the other is physically impossible, unless it is identical: unless the scalar product vanishes. (In quantum mechanics these statements are of course understood in terms of expectation values.)

The very first observation of PNC was made by Wu et al. (1957)[2] in an experiment involving the β-decay of Co^{60}:

$$Co^{60} \rightarrow Ni^{60} + e^{-} + \nu_e. \tag{2}$$

The basic idea is that the scalar product of the nucleus spin (an axial-

2. PNC may have been unwittingly observed earlier by Cox and his collaborators in 1927: they attempted to polarize a beam of β-particles by double scattering, an electromagnetic effect due to spin-orbit coupling (though this mechanism was not understood until 1929). They observed a small polarization, which they put down to scattering. However, their apparatus was not, it was realized soon after, appropriate for producing the effect, and though experimental error makes the matter uncertain, with hindsight it is plausible that they were instead seeing polarization due to PNC in the production of their β-particles. See Maglich 1973 for further discussion of this experiment, Wu's, and others that occurred almost simultaneously.

vector) and electron momentum in the nucleus rest frame (a vector) is a pseudoscalar, and so suitable for testing PNC: unless it vanishes parity is violated. In practice, to measure the expectation value, Wu used a magnetic field to orient a large number of nuclei spins in the same direction, and measured the average electron momentum by counting the relative numbers emitted parallel and antiparallel to the spin. The observed preference for parallel emission means an average electron momentum in the direction of spin, and hence a positive scalar product, demonstrating PNC. More intuitively, one can see from Figure 1 that the mirror image processes are not identical, so if one is possible then the other is not: the laws of nature permit an enantiomorphic process of just one handedness.

Now we turn to consider the kind of physical theory that leads to this situation. Many texts introduce the concept of parity (e.g., Ballentine 1990, 244–250) so we will be satisfied by the following facts. The parity—or reflection—operator, $\hat{P}$, is a unitary and (by convention) Hermitian operator, $\hat{P} = \hat{P}^{-1} = \hat{P}^{\dagger}$, such that: its action on vector quantities (like position and momentum) is $\hat{P}\hat{\vec{V}}\hat{P}^{\dagger} = -\hat{\vec{V}}$; its action on axial vectors (like spin) is $\hat{P}\hat{\vec{A}}\hat{P}^{\dagger} = +\hat{\vec{A}}$; its action on an arbitrary wavefunction is $\hat{P}\psi(\vec{x}) = \psi(-\vec{x})$, so parity eigenstates are even or odd functions, $\hat{P}\pi(\vec{x}) = \pi(-\vec{x})$; and its action on a pseudoscalar like $\hat{\vec{V}} \cdot \hat{\vec{A}}$ is $\hat{P}\hat{\vec{V}} \cdot \hat{\vec{A}}\, P^{\dagger} = \hat{P}\hat{\vec{V}}\hat{P}^{\dagger} \cdot \hat{P}\hat{\vec{A}}\, P^{\dagger} = -\hat{\vec{V}} \cdot \hat{\vec{A}}$.

Strictly then, 'parity nonconservation' means that $d\langle\psi(t)|\hat{P}|\psi(t)\rangle/dt \neq$

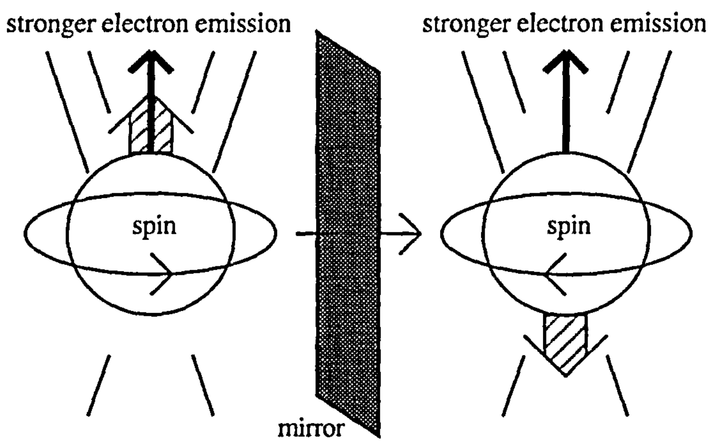

Figure 1. The decay of Co^{60} occurs with a definite handedness—the scalar product of electron momentum (thick arrow) and nuclei spin (shaded arrow) is positive, so the left process is possible but its mirror image is not.

0, a purely quantum phenomenon. More useful (for our purposes) characterizations of PNC are, however, the equivalent statements that $[\hat{H}, \hat{P}] \neq 0$ (where $\hat{H}$ is the Hamiltonian), that $\hat{H}$ fails to be parity symmetric, and that stationary states are not parity eigenstates.[3] For simplicity we shall consider the time-independent Schrödinger equation, and so look at stationary states, rather than time dependent situations, though all the relevant conclusions go through in either case.

In the case of the Wu experiment, it is easy to see that the outcome, analyzed as a stationary state, is not a parity eigenstate. We observe that the (average) scalar product of nucleus spin and electron momentum is positive, so $\hat{P}|\psi\rangle = \pm|\psi\rangle$ leads to absurdity:

$$0 < \langle\psi|\hat{\vec{V}} \cdot \hat{\vec{A}}|\psi\rangle = \langle\psi|\hat{P}^{\dagger}\hat{P}\,\hat{\vec{V}} \cdot \hat{\vec{A}}\,\hat{P}^{\dagger}\hat{P}|\psi\rangle = -\langle\psi|\hat{\vec{V}} \cdot \hat{\vec{A}}|\psi\rangle < 0, \quad (3)$$

where the last step makes use of the action of $\hat{P}$ on a pseudoscalar and the assumption that $|\psi\rangle$ is a parity eigenstate. This example illustrates the importance of observing pseudoscalar quantities, for the absurdity would not arise for a scalar quantity measured on the same system (say the scalar product of nucleus and electron spins). PNC can in this case be explained very directly in the Dirac theory by introducing interaction terms such as $\hat{H}_{PNC} = (\psi^{\dagger}_{p}\gamma^{\mu}\gamma_{5}\psi_{n}) \cdot (\psi^{\dagger}_{e}\gamma^{\mu}\psi_{\nu})$, describing the interaction of a proton, electron, neutrino, and neutron. This term is the four-vector product of an axial-vector and vector (respectively)—overall, a pseudoscalar—so $\hat{P}\hat{H}_{PNC} = -\hat{H}_{PNC}\hat{P}$, and the Hamiltonian is not symmetric (e.g., Bjorken and Drell 1964, Ch. 2).

It will be convenient to have also a simple model of a parity violating interaction: one that has the essential features of naturally occurring processes, but which is more transparent.[4] The simplest kind of case involves particles in one dimension, and the simplest kind of parity violating interaction (with a potential that is bounded from below) is:

$$V(x_1, x_2) = \lambda(x_1 - x_2) + \mu(x_1 - x_2)^2, \quad (4)$$

where x_1 and x_2 are the coordinates of two particles, and λ and μ coupling constants. The asymmetry is manifest, since the potential is neither an odd nor even function. Thus the Schrödinger equation takes the form:

3. A little care is required: even if $[\hat{H}, \hat{P}] = 0$ a stationary state might not be a parity eigenstate if it is a superposition of degenerate energy eigenstates of opposite parity. We will consider nondegenerate Hamiltonians, and ignore this point. Note too that the parity asymmetry of a Hamiltonian is also possible classically: but in that context it does not lead to the violation of a conservation law, because discrete symmetries do not imply conserved quantities classically, only in quantum mechanics.

4. I owe this example and help with it to Tom Imbo.

$$[-\hbar^2/2m(\partial^2/\partial x^2_1 + \partial^2/\partial x^2_2) + \lambda(x_1 - x_2) + \mu(x_1 - x_2)^2]\Psi(x_1, x_2, t) = i\hbar\partial\Psi(x_1, x_2, t)/\partial t. \quad (5)$$

What makes this system a suitably realistic model is that the asymmetry comes from an interaction between the particles, not an external potential.

The solution is fairly straightforward: if one reparameterizes in the variables $y = (x_1 - x_2)$ and $z = (x_1 + x_2)$, and writes $\Psi(x_1, x_2) = \phi(y)\chi(z)$, then with the further substitution $w = y + \lambda/2\mu$ one obtains two equations,

$$[-\hbar^2/4m \cdot \partial^2/\partial z^2]\chi(z) = i\hbar\partial\chi(z)/\partial t \quad (6)$$

and

$$[-\hbar^2/4m \cdot \partial^2/\partial w^2 + \mu w^2 - \lambda^2/4\mu]\phi(w) = i\hbar\partial\phi(w)/\partial t. \quad (7)$$

The first equation clearly describes the free motion of the center of mass of the system, and so can be ignored. The second equation describes the effect of the interaction on the (directed) separation of the particles, y, and so contains the interesting, parity violating, physics. It describes a simple harmonic oscillator in the w-coordinate, and so the stationary state problem has well-known solutions (e.g., Merzbacher 1961, Ch. 5),

$$\phi_n(w) = c_n H_n((4m\mu/\hbar^2)^{1/4}w) \cdot \exp -(\sqrt{m\mu} \cdot w^2/\hbar) \quad (8)$$

with eigenvalues

$$E_n = \hbar\sqrt{\mu/m}(n + 1/2 - \sqrt{m\lambda^4/16\hbar^2\mu^3}), \quad (9)$$

where $H_n(q)$ are Hermite polynomials and c_n normalization coefficients, and $n = 0, 1, 2, \ldots$. To get a feel for what these solutions look like, set $\mu = \lambda = 1/m$ and $\hbar = 1$, and work in the more physical y-variable (remember, $y = x_1 - x_2$). Then, (redefining c_n)

$$\phi_n(y) = c_n H_n(\sqrt{2}(y + 1/2)) \cdot \exp - (y^2 + y). \quad (10)$$

Looking at the first two solutions, and using $H_0(q) = 1$, and $H_1(q) = 2q$, we find,

$$\phi_0 = c_0 \cdot \exp - (y^2 + y) \text{ and } \phi_1 = c_1(y + 1/2) \cdot \exp - (y^2 + y). \quad (11)$$

Under parity, $x \to -x$, so $y = x_1 - x_2 \to -x_1 + x_2 = -y$, so the asymmetry of the stationary states, and thus the parity violating character of the Hamiltonian, is manifest. This asymmetry can be dramatically seen by plotting the wavefunctions and their parity images (Figures 2 and 3).

With an understanding of the phenomena of PNC and the theories behind them, we can now consider how the substantivalist might attempt to parlay this physics into an argument, and then how the relationist might respond.

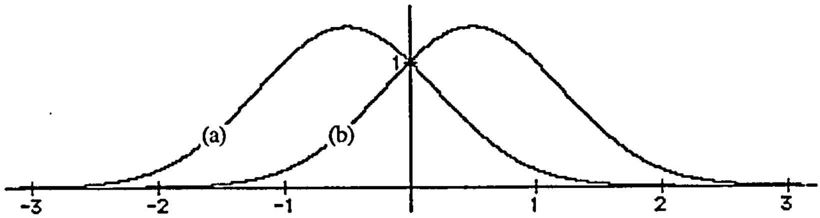

Figure 2. $\phi_1(y)$ (a) versus $\phi_1(-y)$ (b) to show that the first stationary state is not a parity eigenstate—PNC.

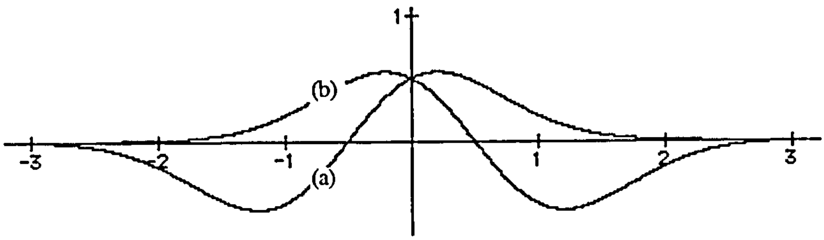

Figure 3. $\phi_2(y)$ (a) versus $\phi_2(-y)$ (b) to show that the second stationary state is not a parity eigenstate—PNC.

5. The Mechanical Challenge and Absolute Structure. In this section we will consider how the substantivalist might attempt to use a successful theory of PNC to support his case, analogous to the use Newton made of his mechanics. First, however, it will be worth roughly locating the general problem facing the relationist. A model for the situation is as if the relationist has to formulate a theory which predicts that in a certain process a hand *of a given handedness* is produced. Of course, the relationist has no problem giving a theory for hand creation, because she can give a relational description of a hand, but there is a difficulty in giving a theory in which it is to be a left rather than right hand, for of course they do not differ in their internal relations. The obvious next step for the relationist is to try to use the hand's external relations, and theorize that the process will produce a hand of a given handedness relative to some external bodies. This strategy worked in defining chirality because we could take whatever enantiomorphs exist contingently and define handedness with respect to them, but in the case of a lawlike account of PNC the same approach will not work: we do not want to tie a law to bodies that exist only contingently, since this undermines its nomic force.

We will continue to pursue this line of thought in parallel to the arguments concerning inertia: (a) we have an observed phenomenon—then nature's preference for inertial motion, now nature's preference for one of a pair of incongruent counterparts; (b) we considered a theory for it—then Newtonian mechanics, now the theories sketched in the previous section; (c) the substantivalist—in this section—argues that the theory requires 'absolute' structure—then affine structure, now an orientation—and that this structure speaks for substantival space; (d) and finally, the relationist considers whether she can construct an acceptable alternative—a relationist theory of PNC analogous to those we considered in §2. By following this plan we will not only have the best possible map to follow, but we will also see Newton's argument from a new perspective.

We shall thus follow Newton closely, considering a series of models, analogous to those considered by Newton in the *Scholium*. The idea is the same as spelling out the globes: to see what commitments of the substantivalist interpretation the relationist must relinquish.

Case 1: Let two Wu experiments be carried out at any two locations, and the results brought together for comparison; they will agree (still assuming an orientable space, and bracketing the fact that the outcome is statistical). In other words, according to the theory the chirality of the law is *universal* in character. Presently, we will postulate spatial structure—an orientation—to ground the law, and so this example is aimed at establishing its homogeneity, as Newton postulated the structure of absolute space to be uniform and unchanging.

At this point we should briefly discuss the assumption of orientability; after all, there is debate about what can be inferred from the observed violation of symmetries even given assumptions about the universality of laws (e.g., Earman 1971), and the topology of space is an empirical matter, currently investigated by studying background radiation (for references see Luminet et al. 1999). First, Geroch (1968) argues that CPT symmetry (invariance under the combined operations of charge conjugation, parity, and time reversal) follows from general assumptions about Lorentzian spacetimes but the observed (directly or indirectly) failure of any other combination of C, P and T to be a symmetry means that spacetime is orientable—admits a 4-form—but that spacelike hypersurfaces need not be. Thus, even if space is not orientable, there is the prospect that the laws require spacetime to have an orientation: our work here then considers the consequences in a simpler setting.

Second, we assume orientability because, as we shall argue below, formulating parity violating laws in local spacetime terms requires the introduction of a preferred orientation, something that is impossible in nonorientable manifolds. However, even if space is not orientable, one can still find a region around any point that is orientable, and parity violating laws can be formulated in the region in terms of a region-bound orientation. It becomes rather mysterious how a universal theory is to be consistently composed of such regional theories, but on the usual assumption that spacetime theories are to be given in terms of local geometric objects, it seems inevitable that a local sense of handedness must be given. In this case, the theory still requires spacetime structure, and it is that fact, not orientability of the whole of space, which does the work in the substantivalist's argument. These considerations also help pin down the force of the thought-experiment; it is not strict universality that is crucial to the argument, but strict correlation of outcomes at different (possibly spacelike separated) locations that requires explanation, and which suggests some homogeneous structure to spacetime, at least over a finite region. Finally, the failure of space to be orientable would offer the relationist no obvious succour in the debate, since her approach suggests no particular way of formulating a parity violating theory in a nonorientable space. Thus, with these qualifications in mind we will continue with our assumption of orientability, leaving further investigation of this assumption for another place.

Case 2: Now we consider again carrying out two Wu experiments, this time in sealed labs that are exact mirror images—and which are enantiomorphs. After the experiment, when the results are compared, according to the theory they will agree: the processes will stand in different relations to the labs in which they occur. In other words, *local relations*—of objects in the lab—do not determine the chirality of the outcome.

The role of this thought-experiment is analogous to that of Newton's bucket. There the idea is—contrary to Descartes—that 'true' motions (distinguished by inertial effects) are not motions relative to immediate surroundings. Put another way, according to Newtonian mechanics, relative motions do not determine whether an object is absolutely rotating. In the present case, local relations do not determine which handedness a process will have.

Case 3: This time consider a world in which the only body is a single Co^{60} nucleus. Relative to the theory, there are apparently two distinct outcomes, one of which is possible and one of which is not: the law specifies a particular relative orientation of nucleus spin and electron momentum expectation value, and forbids its opposite.[5] This thought-experiment is

5. We think here of the 'outcome' as the unitary evolution of the electron wave-function; it has momentum expectation value either parallel or antiparallel to the spin. Obviously, the law is compatible with finding an electron in either direction on observation; one direction is just more likely.

supposed to show that according to the theory no *external relations* at all determine the chirality of the outcome, since there are no external bodies.

Superficially, this example is analogous to the globes experiment, showing that two situations distinguished by the theory are not distinguished relationally, but there are some differences. Most importantly we have to distinguish here between two kinematically possible cases—left versus right—only one of which is dynamically possible, whereas the globes experiment distinguishes two dynamically possible cases—rest versus rotation. This means that the globes argument as formulated earlier does not carry over straightforwardly to the present case: it is far worse to be incomplete with respect to the dynamical possibilities of a successful theory than merely its kinematic possibilities. Indeed, it seems likely that the relationist will want to formulate a theory according to which these two possibilities are in fact only one, as the relationist account of chirality denies two separate handednesses for a lone enantiomorph.

Case 4: Imagine two (nonenantiomorphic) worlds exactly alike until the Wu experiment, which is the first instantiation of the weak interaction. Further, suppose that they realize different outcomes, because the law has a different handedness in the worlds. The point of this example is that relations before the experiment cannot determine the outcome—though they do distinguish the worlds after the experiment—and hence any relational theory must face *indeterminism*. No Leibniz relational theory can say which way the result will turn out before the fact, and given case 1, the fact of which way it turned out must affect every other weak interaction, no matter how far away or how soon after.

Newton argued similarly that since relations could not determine the sense of the rotation of a pair of globes, no relational theory could predict whether a force applied to opposite faces of the globes would increase or decrease the tension (i.e., whether it would increase or decrease the angular velocity). Thus the globes reveals a directly analogous indeterminism in relationist mechanics with respect to Newtonian mechanics.

Before we ask the relationist to respond to these challenges we should see more precisely what absolute geometric structures are implicated in the toy and weak theories.

For the toy theory, it is of course the asymmetric potential term $V_A = \lambda(x_1 - x_2)$ that involves an orientation. Since $(x_1 - x_2) \neq -(x_1 - x_2)$, the Hamiltonian does not depend only on the relative separation of x_1 and x_2, but also on their orientation: on whether $x_1 < x_2$ or $x_1 > x_2$, in absolute terms, not just relative to some arbitrary coordinates. Thus, not until an 'arrow of space' is given is the theory well-defined. This arrow can tell us for two points whether their separation is positive or negative—which is the 'earlier' spatially speaking—and hence give definite meaning to the

Hamiltonian of the theory. (Note the analogy with Newton's first law: 'constant motion', and thus the law, is ill-defined unless some notion of affine structure is given.) Of course, once we have observed the development of the particles we could determine the direction of the arrow, and could express its direction in relational terms, say by two standard objects and their order. Once again, the relationist is not faced with a descriptive problem—or even an epistemological problem—but with formulating a theory of the process in suitable relational terms, and a plausible theory should not make fundamental reference to a contingent standard.

We can make the point about absolute structure more rigorously, and perhaps more forcefully, in the terms of differential geometry, where, recall, orientability is the existence of a nowhere vanishing n-form. In the relevant case of metric spaces, this object takes a canonical form (locally), the 'volume element', vol: vol $\equiv \pm\, e_1 \wedge \ldots \wedge e_n$, where $\{e_i\}$ is any orthonormal basis for the dual to the tangent space (i.e., the covector space) and $\wedge$ is the exterior product for p-forms (e.g., Baez and Munian 1994, 82–87). The choice of sign allows for two possible *orientations*, corresponding (locally) to coordinate axes of opposite senses; in the one-dimensional case, an orientation simply is a choice of $\pm$ a normalized 1-form (covector), which, via the metric, immediately gives a vector field, the arrow of space.

Now it is easy to see the dependence of the asymmetric part of the potential, V_A, on vol. If a curve $\gamma(\tau)$ in $\mathfrak{R}$—the space of the toy theory—takes the values $\gamma(\tau_1) = x_1$ and $\gamma(\tau_2) = x_2$ and has tangent $\gamma'(\tau)$, then, with metric $g(\cdot, \cdot)$, V_A is given by:

$$V_A = \lambda \cdot \left| \int_{\tau_2}^{\tau_2} \sqrt{g(\gamma'(\tau), \gamma'(\tau))} d\tau \right| \cdot \mathrm{vol}(\gamma'(\tau_1))/|\mathrm{vol}(\gamma'(\tau_1))|. \qquad (12)$$

The first term is just the distance between the two points, and the second is ± 1 depending on whether γ runs parallel or antiparallel to the arrow of space (in one-dimension vol is a covector, and so maps vectors into the reals). Thus the necessity of the orientation is manifest, for without it vol is ambiguous between positive and negative; when the law is written in this coordinate-free formulation, the choice of $\pm$vol chooses the sign in the Hamiltonian. Further, under the parity transformation, vol remains unchanged—determining the Hamiltonian—while all other objects are reflected. In this case, $\gamma'(\tau)$ transforms to $-\gamma'(\tau)$ but the metric is unchanged, leading to the desired overall change of sign.

At this point it is worth noting for clarity that there is also a conventional aspect to such handed theories. For suppose the arrow of space now runs in the opposite sense; if V_A remains the potential, then it will have the opposite handedness in space (compared to the original, or compared to some external bodies) and the system will behave differently. But if the potential also changes, $V_A \rightarrow -V_A$, then of course the dynamics will be as before. Thus, it does not make sense in this situation to ask in which direction the arrow of space runs, independently of a given Hamiltonian, and likewise it makes no sense to ask which sign of V_A is correct, independently of an arbitrary choice of arrow. Thus the two possible arrows and two possible Hamiltonians only allow two distinct theories not four. This point acknowledged, we can talk of *the* arrow and *the* Hamiltonian and bear in mind the freedom this actually leaves.

It is also easy to see how this absolute structure plays a role in the thought-experiments of the previous section: assuming the continuity of the Hamiltonian and arrow of space, all interactions governed by the toy theory are identical; it is the vector field, which—like a connection—is taken to be absolute, that determines the handedness of a pair of points, so the form of the interaction is independent of the relations of external bodies; and finally, if the Hamiltonian holds at all times, whether any particles are actually interacting, then it is perfectly determinate, before any interactions, what chirality the first will have.

In the case of the Wu experiment it is the axial-vector, $\Psi^\mu \equiv (\psi^\dagger{}_p \gamma^\mu \gamma_5 \psi_n)$ in the Hamiltonian that leads to PNC. The need for an orientation to define such an object is fairly immediate, once we take into account the fact that Ψ^μ is a 4-vector in Minkowski spacetime, but an axial-vector with respect to 3-dimensional parity transformations. First, a spatial parity operation is defined with respect to a particular foliation of spacetime; spatial reflections transform an instantaneous arrangement of objects. We will take the spacelike hypersurfaces to be orthogonal to an inertial trajectory, and make things easy by working in terms of components in the adapted inertial coordinates. We can decompose Ψ^μ into parts normal and tangent to the foliation: $\Psi^\mu = (\Psi^0, \Psi^i)$, $i = 1, 2, 3$. Further, we can take the restriction, h_{ij}, of the Minkowski metric $\eta_{\mu\nu}$ to each of the surfaces, and thus obtain the spatial volume form, $\mathrm{vol}_{ijk} = dx_i \wedge dx_j \wedge dx_k$. Then, the general expression for an axial 4-vector is

$$A^\mu = (c \cdot \mathrm{vol}_{ijk} v_1^i v_2^j v_3^k,\ h^{j_1 k_2} h^{j_2 k_2} h^{ik_3} \mathrm{vol}_{k_1 k_2 k_3} w_{j_1 j_2}), \tag{13}$$

where c is an arbitrary constant and $w_{j_1 j_2}$ an arbitrary 2-form, and v_j^i are the components of unit vectors along the coordinate directions. The necessity of giving vol an orientation to avoid ambiguity is again evident, as is the axial character of the vector: under parity the volume element is unchanged, as are the metric and any constant and any 2-form, leaving the spatial part unchanged, but $\vec{v} \rightarrow -\vec{v}$, so the timelike component changes sign.

Finally, having shown that the theories require the introduction of an orientation, and hence 'absolute structure', we need only complete the substantivalist's argument. To paraphrase Earman (1989, 125) again, the 'absolutist' asserts that "the scientific treatment of motion . . . requires some absolute quantities . . . such as handedness. To make these quantities meaningful requires the use of an orientation, and this structure must be a property of or inhere in something distinct from bodies. *The only plausible candidate for the role of supporting the nonrelational structures is the spacetime manifold.*" What is the relationist to say?

6. Relationist Responses. In this final section we shall consider how the mechanical challenge posed by PNC might be met by the relationist. We have to hand all the ingredients: the thought-experiments to flesh out the consequences of the substantivalist theory and three kinds of relationist approach. We shall take them in turn and see how they might respond to the experiments: whether they can accommodate them, or whether they would have to diverge from the given theory.

(a) In the Machian approach, locally observed absolute spacetime structures are reinterpreted as phenomenological consequences of a theory involving long range interactions of matter—typically, they are due to the arrangement of the 'fixed stars'. Supposing, very plausibly, that the matter of the universe is distributed enantiomorphically, then this strategy is always possible in principle for a theory of PNC. For example, in the toy theory, if we suppose that in addition to the quantum particles there are three classical bodies—the 'fixed stars'—that are not equally separated, then one can define an arrow of space running from the most remote body towards the other two, and stipulate that it gives the sense of direction required by the Hamiltonian. In higher dimensions, the enantiomorphism of matter also allows a relational distinction between left- and right-handed processes, and thus it is again possible that a relational theory could allow one rather that the other. We can get a better feel for such an approach by considering how it would respond to the substantivalist's thought-experiments.

First, it would probably not entail the agreement of PNC experiments in cases 1–2, since the outcomes would depend somehow on the contingent arrangement of matter. For example, in the toy theory, if the center body moved away from its nearer neighbor and closer to the other body, then, according to the given rule, the arrow of space would reverse producing the opposite outcome for subsequent experiments. The analogy is with Mach's suggestion that, with a really thick bucket, Newton's bucket experiment might turn out differently. The example also reveals that however the matter distribution determines an orientation (locally) it is subject to change, and so the theory must be made consistent with the observed stability of PNC experiments (Mach made this point with regard to local inertial structure). Thus, orientation must be relatively insensitive to local variations in matter distribution; on the other hand, the cosmological hypothesis of homogeneity and isotropy implies that the universe is not enantiomorphic on large scales, and so an orientation could not depend on large scale matter distributions either.

A referee suggested that the situation here is quite grim for the Machian: the problem being not so much the instability of an orientation but the difficulty in discovering the relevant one. In the case of inertia, the universe's center of mass frame provides a natural standard (although there is still latitude in the choice of a standard of zero rotation, for which Mach's choice is only one possibility). In particular, any (sufficiently large) subset of the fixed stars with the same center of mass as the whole would presumably lead to (almost) identical local inertial structure (modulo the more sensitive standard of zero rotation). On the other hand, given some rule connecting mirror asymmetry to a particular direction for an arrow of space, different asymmetric collections of the fixed stars should disagree on the arrow as often as they agree. The problem is not that asymmetry could not determine an orientation, but that the inductive base—the local orientation determined in our universe—is far too narrow to start to imagine what the connection is; at least with inertia the rest frame provides a good guess. Mach might have accepted this conclusion, but if one wishes to go beyond cataloguing local contingent spatial structures, the problem must be faced.

What if we consider a single PNC experiment? In the parallel case of the globes, Mach argued that we cannot infer anything about what might be observed, though he proposed that local inertial structure would at least be very different, and perhaps not exist at all. In the present case, one might expect, in the absence of matter, that there would be no orientation so that the theory would break down. However, in the absence of any external matter there are, for the relationist, not two distinct handednesses for the outcome but one; thus one possibility is that a Machian theory simply predicts an enantiomorphic outcome of indeterminate handedness in such a case. Finally, if we take a suitably robust understanding of laws, so that they can be true even when uninstantiated, then no indeterminism threatens (thoroughgoing Machians may be suspicious of such a view of laws, but there is no obvious reason why someone who wants a Machian theory of dynamics should have to buy the whole package of Machian philosophy): the arrangement of matter plus the laws may determine how the first experiment will turn out. (And if the theory were changed so that the same distribution determined the opposite orientation, then the process would occur with the opposite handedness.)

The approach, as sketched, suffers from the same chronic problem that Machians face in the case of inertial structure: all that is given is an idea for a theory, not a theory itself. It is one thing to point out that an asymmetric matter distribution makes it possible to distinguish two outcomes, but nothing specific has been said about the form of the interaction between bodies that induces an orientation. Especially, we are not given enough theory to tell how substantially different arrangements would change the interaction: would its form as well as sign be changed? It is unclear how the Machian might even begin to answer these questions.

(b) The second relationist strategy—van Fraassen's—involves denying that absolute structures supervene on relations. The idea is that a theory is given by dynamical equations, plus the stipulation that 'the equations hold in some frame(s)', defined in a relationally respectable way (§2). The approach accepts that there are absolute structures, but denies that spacetime is the 'only plausible candidate' for supporting them, instead claiming that absolute structures inhere in relational frames. They are features of the way things move relationally, but the relations at a time need not determine the structures, only motions over time may do that (of course, empirically, both relationist and substantivalist are restricted to observing the relative motions of bodies to determine absolute structures, and so are in the same position regarding epistemic access to the structures). I have discussed the general plausibility or otherwise of such views in my 1999a, so we will not rehearse those arguments here; instead we should see how such an approach might react to our thought-experiments.

We can easily convert the toy theory to the approach: one of the external bodies can be used to define an adapted frame and its mirror image—a single axis running in one direction or the other. Next we give the theory by writing down the Hamiltonian and stipulating that the corresponding Schrödinger equation holds in one frame or the other. For the weak interaction, we construct a three-dimensional frame adapted to some body, and stipulate that our Hamiltonian holds in that frame or its mirror image. (Note that constructing such adapted frames requires rigid bodies whose parts can be reidentified over time; if all matter is quantum then even the approximate realization of these conditions requires that some matter exist in a classical limit state.) Note that this approach does not postulate that the reference body itself has some dynamical effect on the interaction, along the lines of Neumann's 'body-α' for example. We have complete freedom to define the frames with respect to any reference body whatever: it is the family of frames that is dynamically relevant.

The frame is defined globally on the space so PNC experiments will agree universally, and since the theory is given with respect to arbitrary reference bodies, the outcome is indeed independent of local relations. The case of a PNC process in the absence of external bodies is more subtle, since there is no way to define an adapted reference frame and formulate the theory (in contrast to the case of the globes, in which one body itself could define a frame) if there is no (semi-) classical matter. Strictly speaking then, Case 3 cannot be a model of the theory at all, which seems a rather unfortunate consequence, since it suggests that external bodies have a dynamical effect, contrary to the intentions of the approach. Finally, there is no indeterminism, again assuming that the Schrödinger equation holds in some determinate frame even before it is instantiated.

(c) The final relationist approach—Sklar's—involves adding new spatiotemporal properties to the standard Leibniz relations so that absolute dynamical structures are determined by the full set of spatiotemporal properties of bodies. The question then is what new properties might yield an ersatz orientation, in the way that monadic acceleration captures inertial structure. In the toy model, an answer is that distance relations must no longer be taken to be scalar quantities, but pseudoscalars: between any two bodies is defined the directed distance, $\pm|x_1 - x_2|$, the sign depending on which particle is relatively 'to the right' of the other. We shall say that $\pm|x_1 - x_2|$ is the distance 'from' x_1 'to' x_2, and is positive if x_2 lies in the direction of the arrow of space from x_1. Whether this approach can be extended to higher dimensions or the weak interaction we will leave as an open question, having no suggestions either way. Instead we shall consider the thought-experiments one more time, focusing on the toy theory.

First, asking whether outcomes will agree universally brings up an important feature of this kind of theory, namely that there must be long-range correlations between the new relations. For example, if the distances from A to B and from B to C are positive, then the distance from A to C is positive. In the case of inertia, similar correlations occur: if A and B move with a constant relative velocity, then their monadic accelerations are equal. Such correlations are interesting in that they manifest a kind of nonlocality (e.g., Budden 1998). If the correlations do occur, then the theory predicts the same outcomes anywhere in space, and so the laws hold in the same sense universally. Second, since the directed distance is not dependent on the configuration of local (external) bodies, we also expect agreement when the experiment is carried out in mirror image laboratories. Third, the theory does make sense in an otherwise empty universe, but without external reference bodies there is no relational way to distinguish the 'two' possible directions of distance, and so there is only one possible outcome. Finally there is no problem of indeterminism for the theory, because the prior configuration of matter, including the fact about which direction is positive, and the prior law determine which outcome will occur.

In conclusion, we have seen that formulating parity violating laws requires the introduction of absolute spacetime structures, in a way quite analogous to Newton's Laws, and that they thus foster substantivalism in just the same way that Newtonian mechanics does. Not surprisingly then, the arguments on both sides of the substantival-relational debate proceed quite similarly for PNC as of regular mechanics, and if we have not taken a strong line on whether PNC settles the debate, it is because one's view of the matter is likely to depend strongly on one's view of Newton's arguments. That said, the work done here has shown only in general terms how PNC bears upon the debate, and future work may show that, in the details, PNC is more decisive.

Finally, reformulating Newton's arguments in the context of PNC reveals a crucial point that various commentators (especially Stein 1967) have emphasized; that they are not based on direct inductions, but by assuming the theory that accommodates them. Newton does not argue that since the true rotation of the water is not rotation relative to the bucket, it must be relative to absolute space, nor that since observed rotating bodies exhibit tension, they also would in empty worlds. Instead, he explained the consequences of the best account of the observed phenomena, to show the problems with giving it a relational interpretation. The relationist cannot honestly meet this challenge with skepticism about the inferences, but by producing his own theory. This paper has been written with this model of Newton's arguments in mind: first we understood the relevant phenomena, then, assuming suitable theories, we considered various models that cause problems for the relationist. In this final section we have considered programmatically how the relationist might try to meet this challenge.

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